DELHI PUBLIC SCHOOL, JAMMU
SAMPLE QUESTIONS FOR PRE-BOARD EXAMINATION
(As per the pattern of CBSE Sample Paper)

(2019-20)

Sub: Mathematics **Class: XII**

Q1. Show that the relation R in the set Z of integers given by R = {(a, b) : 2 divides a – b} is an equivalence relation. Also find its all possible equivalence classes.

Q2. Check whether the relation R in R of real numbers defined by R = {(a , b) : a $\leq$ b^3} is reflexive, symmetric or transitive.

Q3. Show that the relation R defined by (a, b) R(c, d) $\Leftrightarrow$ a + b = b + c on the set N x N is an equivalence relation.

Q4. F : R $\rightarrow$ R, g : R $\rightarrow$ given by f(x) = [x], g(x) = $|x|$, then find (fog) $\left(\frac{-2}{3}\right)$ and (gof)$\left(\frac{-2}{3}\right)$.

Q5. Show that the function f : N $\rightarrow$ N defined by f(x) $\begin{cases} \frac{n+1}{2}, n is\ \ odd \\ \frac{n}{2}\ , nis\ even \end{cases}$ is not bijective.

Q6. Consider f: $R_+ \rightarrow [-5, \infty)$ given by f(x) = $9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$

Q7. Discuss the commutativity and associativity of the binary operation * defined on Q by the rule a* b = a – b + ab for all a, b, c $\in$ Q.

Q8. Let R^+ be the set of all positive reals. Define an operation 'o' on R^+ by a o b = $\frac{ab}{4}, \forall$ a, b $\in R^+$. Show that the operation 'o' is commutative as well as associative. Also find the identity element and the inverse of a.

Q9. Find the value of $\cos^{-1}\left(\cos\frac{4\pi}{3}\right)$.

Q10. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{2}\right)$

Q11. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}cos^{-1}\ x, -\frac{1}{\sqrt{2}} \leq x \leq 1$

Q12. Prove that $\tan^{-1}\frac{1}{3} + tan^{-1}\frac{1}{7} = \frac{\pi}{4}$.

Q13. Prove that $\cos^{-1}\frac{12}{13} + sin^{-1}\frac{3}{5} = sin^{-1}\frac{56}{65}$.

Q14. Solve : $\tan^{-1}\frac{x-1}{x-2} + \ tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$.

Q15. If A and B are symmetric matrices, determine whether AB – BA is symmetric or skew – symmetric matrix.

Q16. Express $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as a sum of symmetric and skew-symmetric matrices.

Q17. Obtain the inverse of the matrix A = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ using elementary operations.

Q18. There are three families A, B and C. The number of men, women and children in these families are as under:

	Men	Women	Children
Family A	2	3	1
Family B	2	1	3
Family C	4	2	6

Daily expenses of men, women and children are Rs.200, Rs.150 and Rs.200 respectively. Only men and women earn and children do not. Using matrix multiplication, calculate the daily expenses of each family. What impact does more children in the family create on the society ?

Q19. If A is a square matrix of order 3 x 3 such that $|A| = 5$, then find.$|4A|$

Q20. If A is a square matrix of order 3 x 3 such that $|A| = 5$, then find $|adjA|$.

Q21. If x, y, z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then show that 1 + xyz = 0.

Q22. Show that $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(xy+yz+zx)$.

Q23. Prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$

Q24. Prove that $\begin{vmatrix} x-y-z & 2x & 2x \\ 2y & y-z-x & 2y \\ 2z & 2z & z-x-y \end{vmatrix} = (x+y+z)^3$.

Q25. If $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$, then prove that a + b + c = 0 or a = b = c.

Q26. Solve the system of equations by using matrix method :

$$2x + 3y + 3z = 5$$
$$x - 2y + z = -4$$
$$3x - y - 2z = 3$$

Q27. Solve the following system of equations by matrix method where x, y, z ≠ 0.

$$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10$$

$$\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$$

Q28. Use the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations

x – y + 2z = 1

2y – 3z = 1

3x – 2y + 4z = 2

Q29. Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school wants to award Rs.x each, Rs.z each for the three respective values to 3, 2 and 1 students respectively with a total award money of Rs.1600. School B wants to spend Rs.2300 to award its 4, 1 and 3 students on the respective values(by giving the same award money to the three values as before). If the total amount of award for one prize on each value is Rs.900, using matrices, find the award money for each value. Apart from these three values, suggest one more value which should be considered for award.

Q30. Show that the function *f* defined by f(x) = $\begin{cases} \frac{|x-4|}{x-4}, x \neq 4 \end{cases}$ is continuous every where except at x = 4.

Q31. Determine the value of *k* for which the function f(x) = $\begin{cases} \frac{1-\cos 2x}{2k^2}, x \neq 0 \\ k, x = 0 \end{cases}$ is continuous at x = 0.

Q32. Determine the values of a, b and c for which the function f(x) = $\begin{cases} \frac{\sin (a+1)x + \sin x}{x}, x < 0 \\ c, x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}}, x > 0 \end{cases}$ is continuous at x = 0.

Q33. Prove that the function f given by f(x) = $|x-1|, x \in R$ is continuous at x = 1 but not differentiable at x = 1.

Q34. Find $\frac{dy}{dx}, if\ y = (\sin x)^{\cot x} + (\sin x)^{secx}$.

Q35. Find $\frac{dy}{dx}, if$ $x^y + y^x = a^b$.

Q36. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then show that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

Q37. If y = $\sin^{-1}\left(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right)$ $and\ 0 < x < 1$, then find $\frac{dy}{dx}$.

Q38. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for -1 < x < 1, prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$.

Q39. If cos y = xcos (a + y), with cos a $\neq \pm 1$, prove that $\frac{dy}{dx} = \frac{cos^2(a+y)}{\sin a}$.

Q40. If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

Q41. Find the derivative of $\log_e(\sin x)$ w.r.t $\log_e(\cos x)$

Q42. If x = a (cost + t sin t) and y = a (sin t – t cost), find $\frac{d^2y}{dx^2}$.

Q43. If y = $(tan^{-1}x)^2$, show that $(x^2 + 1)^2 y_2 + 2x (x^2 + 1) y_1 = 2$.

Q44. If y = $\sqrt{x+1} - \sqrt{x-1}$, prove that $(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - \frac{1}{4}y = 0$

Q45. Verify Rolle‟s Theorem for the function $y = x^2 + 2$ in the interval [-2, 2].

Q46. Use Lagrange‟s Mean Value Theorem to determine a point on the curve y = $\sqrt{x-2}$ at the tangent is parallel to the chord joining the points (2,0) and (3,1).

Q47. A man 2metres high walks at a uniform speed of 5km/hr away from a lamp post 6 metre high. Find the rate at which the length of his shadow increases.

Q48. A ladder 5m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 m/s. How fast is the height on the wall decreasing when the foot of the ladder is 4 m away from the wall.

Q49. Sand is pouring from a pipe at the rate of 12 cc/sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand-cone increasing when the height is 4 cm.

Q50. A particle moves along the curve y = $\frac{2}{3}x^3 + 1$. Find the point on the curve at which y – coordinate is changing twice as fast as x-coordinate.

Q51. Prove that the function f(x) = $\frac{x^3}{3} - x^2 + 9x, x \in [1,2]$ is strictly increasing. Hence find the minimum value of f(x).

Q52. Show that the line $\frac{x}{a} + \frac{y}{b} = 1$ touches the curve y = $be^{-\frac{x}{a}}$ at the point where the curve crosses the y-axis.

Q53. For the function f(x) = $-2x^3 - 9x^2 - 12x + 1$, find the intervals in which f(x) is i) increasing ii) decreasing.

Q54. Find the intervals in which f(x) = sin x – cos x, where 0 < x < 2π is increasing or decreasing.

Q55. Find the equation of the tangent and normal to the curve y = $\sin^2 x$ at the point $\left(\frac{\pi}{4}, \frac{3}{4}\right)$.

Q56. Find the point on the curve y = $3x^2 - 12x + 6$ at which the tangent is parallel to the x-axis. Also find the equation of tangent at that point.

Q57. Find the points on the curve y = $x^3 - 2x^2 - x$ at which the tangent lines are parallel to the line y = 3x – 2.

Q58. Prove that the curves x = y^2 and xy = k cut at right angles if $8k^2 = 1$.

Q59. Use differentials to approximate $\sqrt{50}$.

Q60. Find the approximate change in the volume of a sphere of radius 10cm if there is an error of 0.1 cm in measuring its radius.

Q61. Find the absolute maximum and minimum values of the function defined by f(x) = $\frac{1}{3}x^3 - 3x^2 + 5x + 8\ in[0,4]$

Q62. Evaluate: $\int \frac{e^x(1+x)}{cos^2(e^x x)} dx$.

Q63. Evaluate: $\int e^x \frac{x^2+1}{(x+1)^2} . dx$.

Q64. Evaluate: $\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$.

Q65. Evaluate: $\int \frac{(3 \sin \theta - 2) \cos \theta}{5 - cos^2 \theta - 4\ sin\theta} d\theta$.

Q66. Evaluate: $\int \frac{x+2}{2x^2+6x+5} dx$.

Q67. Evaluate: $\int \frac{x^4}{(x-1)(x^2+1)} . dx$.

Q68. Evaluate: $\int \frac{2x}{(x^2+1)(x^2+4)} . dx$.

Q69. Evaluate: $\int \frac{\sin x}{(4+\cos^2 x)(2-\sin^2 x)}.dx.$

Q70. Evaluate: $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}.dx.$

Q71. Evaluate: $\int \frac{1}{x(x^4+16)}.dx.$

Q72. A point on the parabola $y^2 = 18x$ at which the ordinate increase at twice the rate of the abscissa is

a. (2, 6) b. (2, - 6) c. $\left(\frac{9}{8}, -\frac{9}{2}\right)$ d. $\left(\frac{9}{8}, \frac{9}{2}\right)$

Q73. The rate of change of the volume of a sphere w.r.t its surface area, when the radius is 2 cm, is

a. 1 b. 2 c. 3 d. 4

Q74. If there is an error of k% in measuring the edge of a cube, then the percent error in estimating its volume is

a. k b. 3k c. $\frac{k}{3}$ d. None

Q75. A lamp of negligible height is placed on the ground ℓ_1 away from a wall. A man $\ell_2 m$, tall is walking at a speed of $\frac{\ell_1}{10}$ m/s from the lamp to the nearest point on the wall. When he is midway between the lamp and the wall, the rate of change in the length of this shadow on the wall is

a. $-\frac{5\ell_2}{2}m/s$ b. $-\frac{2\ell_2}{5}m/s$ c. $-\frac{\ell_2}{2}m/s$ d. $-\frac{\ell_2}{5}m/s$

Q76. A man is moving away from a tower 41.6 m high at a rate of 2 m/s. If the eye level of the main is 1.6 m above the ground, then the rate at which the angle of elevation of the top of the tower changes, when he is at a distance of 30 m from the foot of the lower, is

a. $-\frac{4}{625}rad/s$ b. $-\frac{2}{25}rad/s$ c. $-\frac{1}{625}rad/s$ d. None

Q77. The radius of a right circular cylinder increases at the rate of 0.1 cm./min and the height decreases at the rate of 0.2 cm/min. The ratio of change of the volume of the cylinder, in cm^3/min, when the radius is 2 cm and the height is 3 cm is

a. – 2p b. $-\frac{8\pi}{5}$ c. $-\frac{3\pi}{5}$ d. $\frac{2\pi}{5}$

Q78. If f(x) and g(x) are differentiable functions for $0 \le x \le 1$ such that f(0) = 10, g(0) = 2, f(1) = 2, g(1) = 4, then in the interval (0, 1)

a. $f'(x) = 0\ for\ all\ x$ b. $f'(x) + 4g'(x) = 0$ for at least one x

c. $f'(x) = 2g'(x)$ for at most one x d. none of these

Q79. A maximum point of $3x^4 - 2x^3 - 6x^2 + 6x + 1$ in [0, 2] is

a. x = 0 b. x = 1 c. x = $\frac{1}{2}$ d. does not exist

Q80. If f(x) = $\begin{cases} 3x^2 + 12x - 1, & -1 \le x \le 2 \\ 37 - x, & 2 < x \le 3 \end{cases}$ then

a. f(x) is increasing in [-1, 2] b. f(x) is continuous in [-1, 3]

c. f(x) is maximum at x = 2 d. all the above

Q81. Let f(x) = $(1 + b^2)x^2$ + 2bx + 1 and m(b) is a minimum value of f(x). If b can assume different values, then range of m(b) is equal to

a. [0, 1] b. (0, 1, 2] c. [1/2, 1] d. (0, 1]

Q82. If x be real then the minimum value of f(x) = $3^{x+1} + 3^{-(x+1)}$ is

a. 2 b. 6 c. 2/3 d. 7/9

Q83. The least value of $2^{(x^2-3)^3+27}$

a. 2^{27} b. 2 c. 1 d. None of these

Q84 The value of the integral $\int_0^\infty \frac{x \log x}{(1+x^2)^2}$ dx is :

(a) 1 (b) 0 (c) 2 (d) none

Q85 The value of the integral $\int_0^1 \cot^{-1}(1 - x + x^2)dx$ is :

(a) π – log 2 (b) $\frac{\pi}{2}$ – log 2 (c) π + log 2 (d) $\frac{\pi}{2}$ + log 2

Q86 The area of the region bounded by y = $|x - 1|$ and y = 1 is :

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) none

Q88 The value of the integral $\int_0^1 \frac{1}{(x^2+1)^{3/2}}\, dx$ is :

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) $\sqrt{2}$

Q89 $\int_0^{50\pi} |\cos|\, dx =$

(a) 100 (b) 50 (c) 0 (d) none

Q90. The value of f(0), so that the function f(x) = $\frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$ is continuous at each point in its domain is equal to:

a. 2 b. 1/3 c. 2/3 d. – 1/ 3

Q91. The value of f(0) so that the function f(x) = $\frac{\sqrt[3]{1+x}-\sqrt[4]{1+x}}{x}, x \neq 0$ becomes continuous at x = 0 is:

a. $\frac{1}{3}$ b. $\frac{1}{4}$ c. $\frac{1}{5}$ d. $\frac{1}{12}$

Q92. The function f(x) = $\frac{1+\sin x-\cos x}{1-\sin x-\cos x}, x \neq 0$ is continuous at x = 0 if (0) is:

a. 1 b. – 1 c. $\sqrt{2}$ d. - $\sqrt{2}$

Q93. If f(x) = $\frac{x^2-bx+25}{x^2-7x+10}$ for x ≠ 5 is continuous at x = 5, then the value of f(5) is

a. 0 b. 5 c. 10 d/ 25

Q94. Let f(x) = $\begin{cases} |x-1|^n \; sin\left(\frac{1}{x-1}\right), & x \neq 1 \\ 0, & x = 1 \end{cases}$ then f(x) is continuous at x = 1 if m belongs to:

a. (0, 1) b. [0, ∞) c. [1, ∞) d. None

Q95. Let f(x) = $\begin{cases} \frac{(x^3+x^2-16x+20)}{(x-2)^2}, & if \; x \neq 2 \\ k, & if \; x = 2 \end{cases}$. if f(x) is continuous for all x, then k is equal to:

a. 2 b. 3 c. 6 d. 7